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# Control Charts or Process Behaviour Charts

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Process Re-Engineering Team

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## Objectives of the Day

Understand the best type of data to collect

Know how to construct, apply and interpret  
process behaviour IMR (XMR) charts.

Learn through hands on exercises

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## Quantitative data

- Numerical data or information, either:
  - **Attributes** data – presence or absence of a feature or characteristic – count, %
  - **Variables** data – measurement of a characteristic – unit of measurement (cm, degrees etc)
- Generally variables data gives more information than attributes data

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## Attributes Data

Non-speeders

Speeders

Set 1



(2)



(4)



Set 2



(2)



(4)

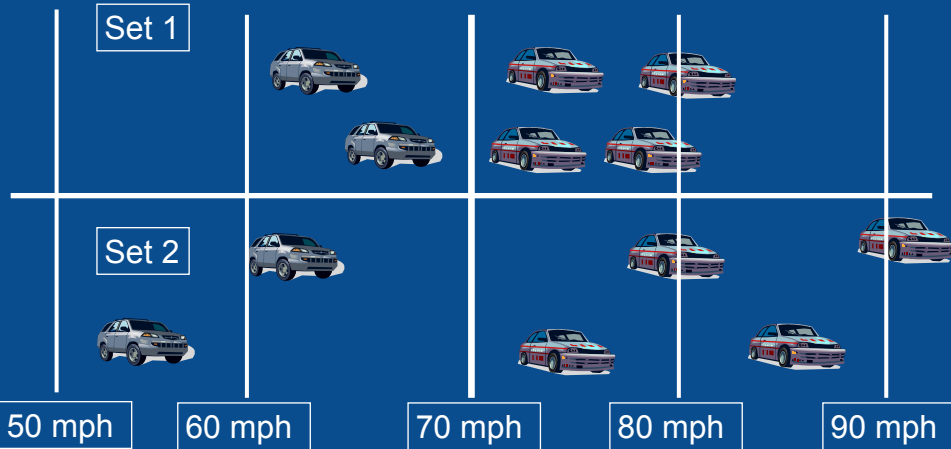


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## Variables Data



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## Why We Need to Measure

Measurement helps us to...

- understand current process performance
- See the impact of changes to the process
- Detect signals of potential problems

Which in turn means that we...

- Make the right decisions
- Take the right action

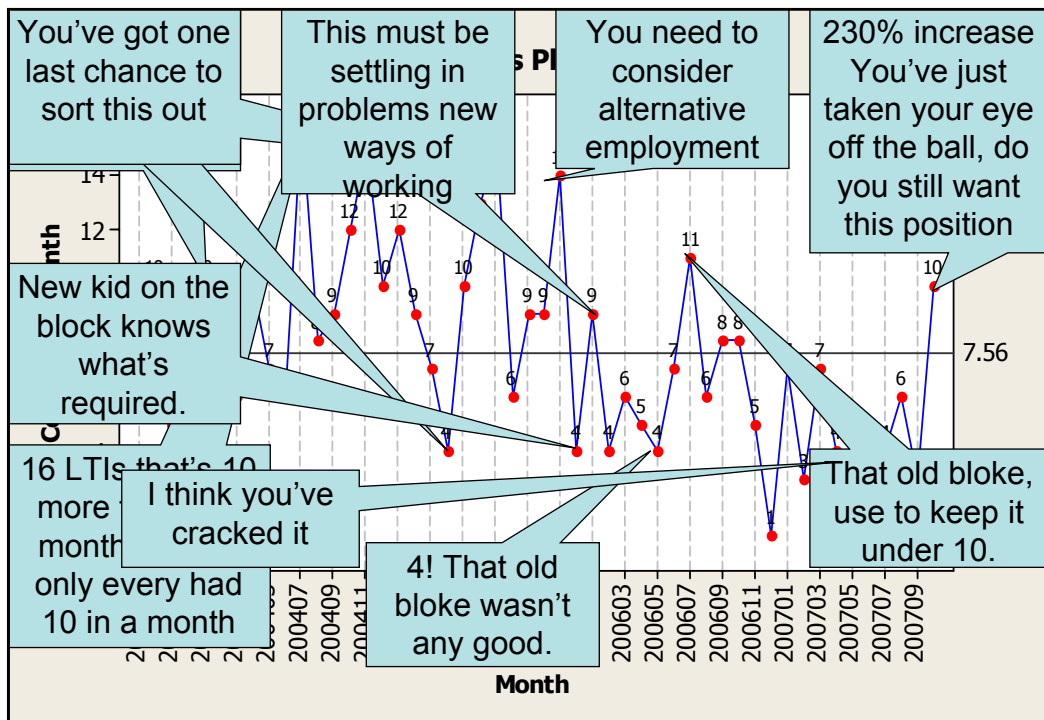
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## Danger - Variation - why so important ?

- See trends when there aren't any
- Try and explain something that isn't there
- Waste time, effort and resources
- Blame and/or credit people for things they have no control over
- We make mistakes because we over and under react
- Can't predict - impossible to plan



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## Danger - Variation

“While every data set contains noise,  
some data sets may contain signals.

Therefore, before you can detect a signal  
within a given data set, you must first filter out  
the noise”

Donald Wheeler (a very famous statistician)

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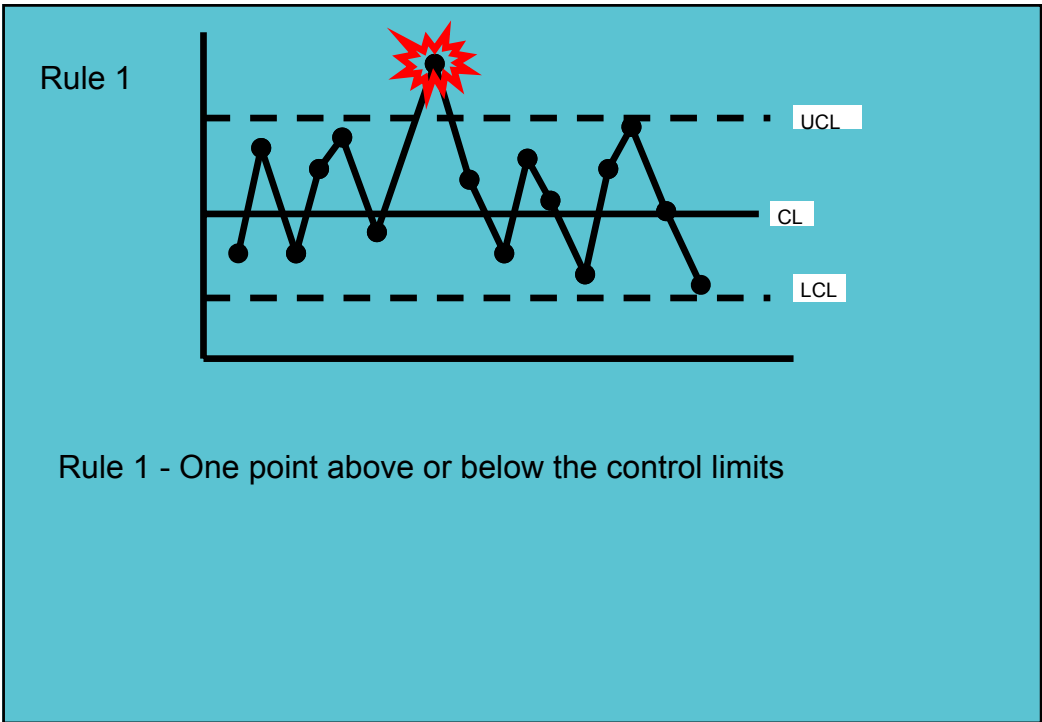
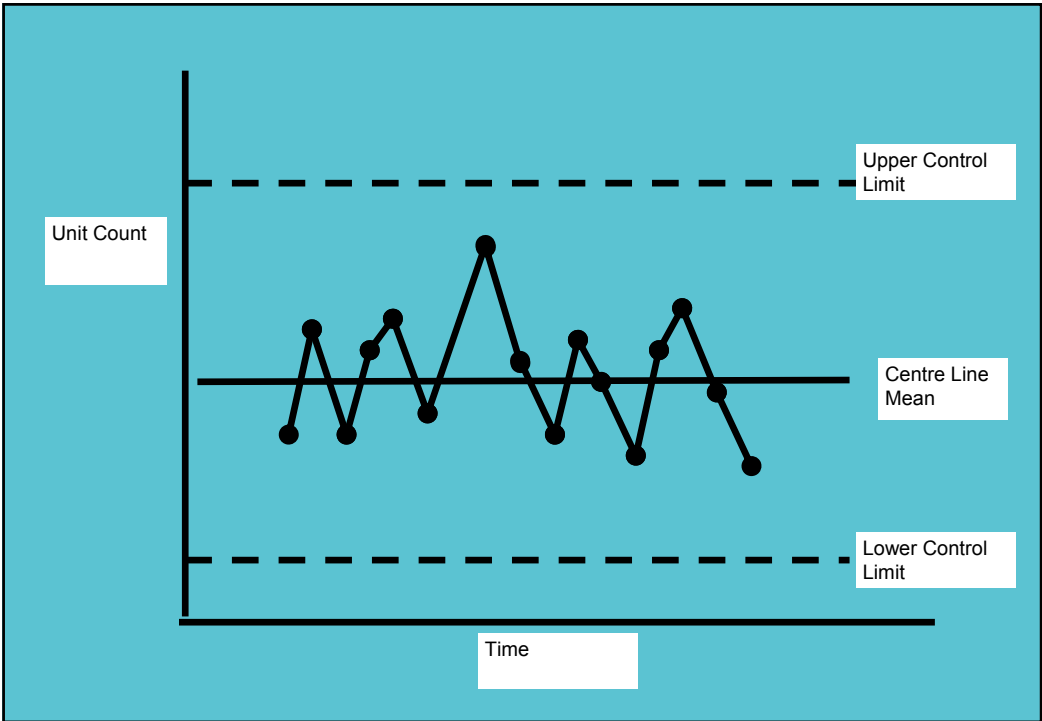


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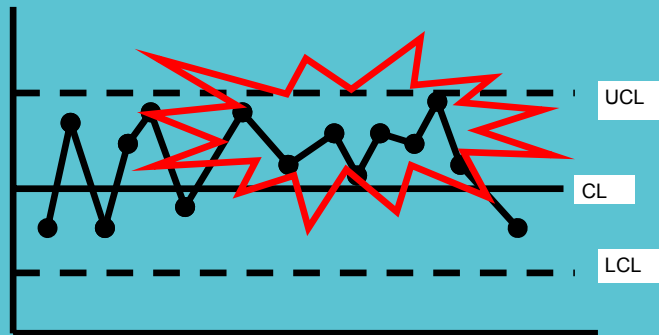
## Two Types of Variation

- Common cause; normal - expected – predictable
- Special cause; assignable - unusual - something different

Process behaviour charts identify variation



Rule 2



Rule 2 - Eight points in a run above or below the centre line mean

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## IMR Charts: Construction Basics

2 charts are required:

- Individual
- Moving range

Data must be in time order

Minimum 12-20 data points [even though our following example uses only 6]

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## IMR Charts: Construction

- Step 1 - Range Between Numbers

Month/year	Oct-04	Nov-04	Dec-04	Jan-05	Feb-05	Mar-05
% late	0	5	15	12	7	24

Moving Range                    5                    10                    3                    5                    17

- Step 2 - Individual Mean

=Sum individual values / number of individual values

Step 3 - Mean Moving Range

=Sum Moving Range values / number of Moving Range values

\*Note number of moving range values is one less than the individual values

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## IMR Charts: Construction

- Step 4 - Control Limits for Individual Chart

Upper Control Limit = Individual Mean + (2.66 x Mean Moving Range)

Lower Control Limit = Individual Mean – (2.66 x Mean Moving Range)

- Step 5 Control Limits for the Moving Range Chart

Upper Control Limit = 3.27 x Moving Range Mean

Lower Control Limit = 0 (always)

- Step 5 - Plot data, means and control limits

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## Exercise 1 (IMR Charts)

Work in groups

[Tip; Make sure you have room for 24 points on X axis]

## Exercise 1 (IMR Charts) Step 1 Determine Moving Ranges

Permit No.	Hours to determine	Range
1.	10.7	-
2.	13.0	
3.	11.4	
4.	11.5	
5.	12.5	
6.	14.1	
7.	14.8	
8.	14.1	
9.	12.6	
10.	16.0	
11.	11.7	
12.	10.6	

**Make sure you have room for 24 points on X axis**

## Exercise 1 (IMR Charts) Step 1 Determine Moving Ranges

Permit No.	Hours to determine	Range
1.	10.7	-
2.	13.0	2.3
3.	11.4	1.6
4.	11.5	0.1
5.	12.5	1.0
6.	14.1	1.6
7.	14.8	0.7
8.	14.1	0.7
9.	12.6	1.5
10.	16.0	3.4
11.	11.7	4.3
12.	10.6	1.1

**Make sure you have room for 24 points on X axis**

## Exercise 1 (IMR Charts)

### Step 2 - Individual Mean

- A) Sum values of the determined hours = 153  
B) Divide by number of values = 12.75

### Step 3 - Mean Moving Range

- A) Sum values of the Moving Range = 18.3  
B) Divide by number of values = 1.66

(Note the number of values is 1 less than that on the  
Individuals)

## Exercise 1 (IMR Charts)

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### Step 4 - Control Limits Individuals

#### A) Upper Control Limit for Individuals

$$= \text{mean} + (2.66 \times \text{Moving Range Mean})$$

$$= 12.75 + (2.66 \times 1.66) = 17.17$$

#### B) Lower Control Limit for Individuals

$$= \text{mean} - (2.66 \times \text{Moving Range Mean})$$

$$= 12.75 - (2.66 \times 1.66) = 8.33$$

(Note: if the Lower Control Limit is less than zero and results can never be below zero then set the LCL at zero)

## Exercise 1 (IMR Charts)

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### Step 5 - Control Limits Moving Range

#### A) Upper Control Limit for Moving Range

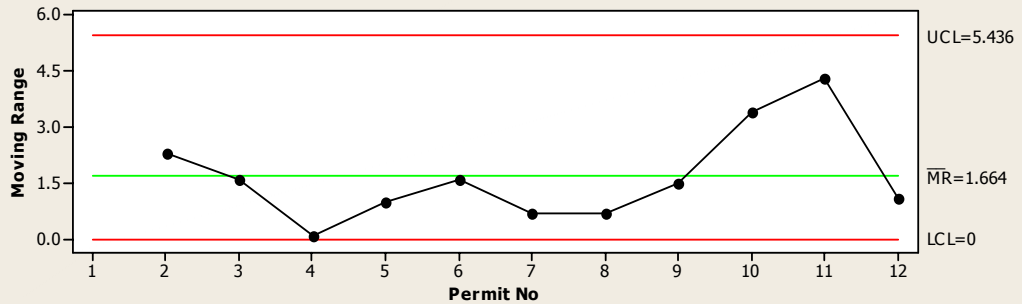
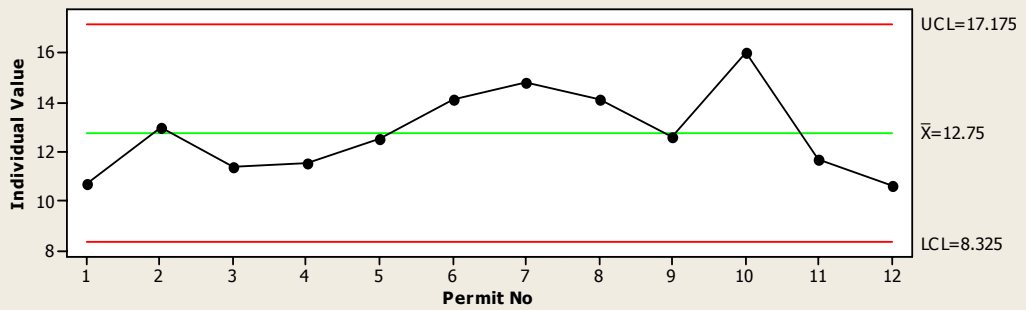
$$= \text{Mean Moving Range} \times 3.27$$

$$= 1.66 \times 3.27 = 5.43$$

#### B) Lower Control Limit for Moving Range

$$= 0$$

## Hours Taken to Determined Water Quality Permits



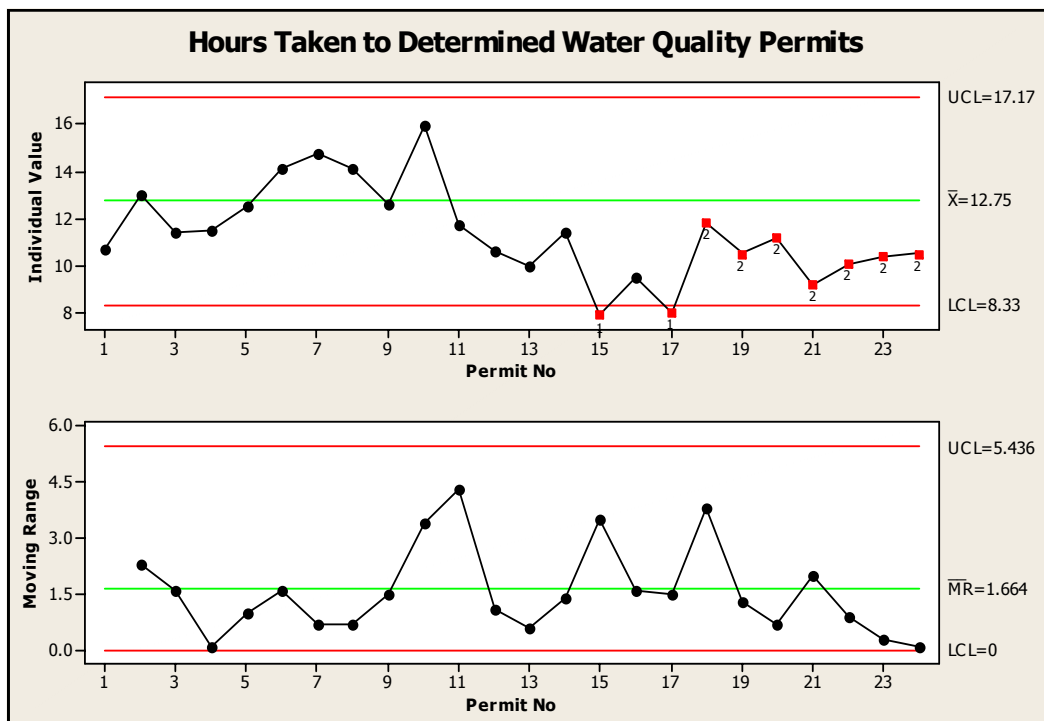
## Exercise 1 (IMR Charts) New Data - Moving Ranges

Permit No.	Hours to determine	Range
13	10	
14	11.4	
15	7.9	
16	9.5	
17	8.0	
18	11.8	
19	10.5	
20	11.2	
21	9.2	
22	10.1	
23	10.4	
24	10.5	

# Exercise 1 (IMR Charts)

## New Data - Moving Ranges

Permit No.	Hours to determine	Range
13	10	0.6
14	11.4	1.4
15	7.9	3.5
16	9.5	1.6
17	8.0	0.5
18	11.8	3.8
19	10.5	1.3
20	11.2	0.7
21	9.2	2
22	10.1	0.9
23	10.4	0.3
24	10.5	0.1

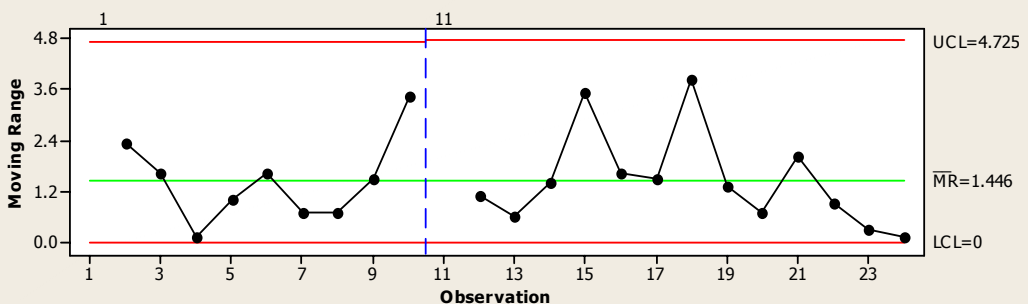
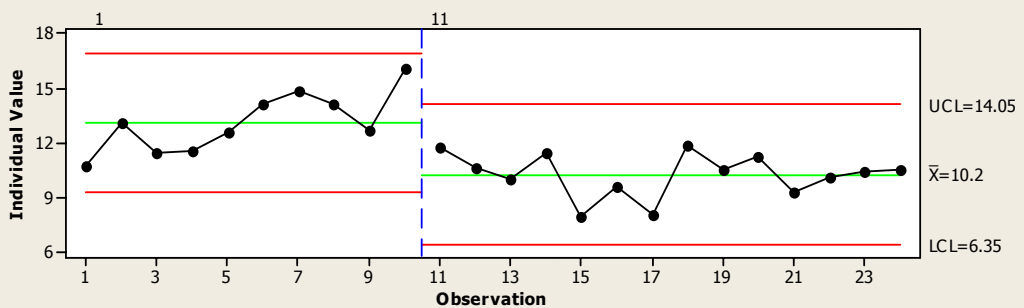


## Re-calculating your control limits

Only do this when:

- Evidence of an assignable cause
- When the cause known
- When the process change desirable
- Or if not a desirable change you can't fix it straight away
- It is sustained

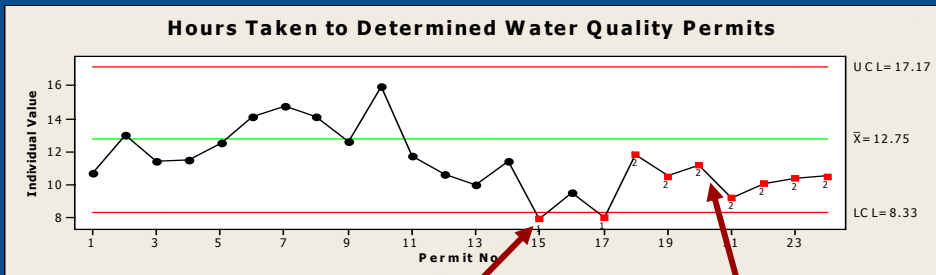
### Hours Taken to Determined Water Quality Permits



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## Individual Chart - summary

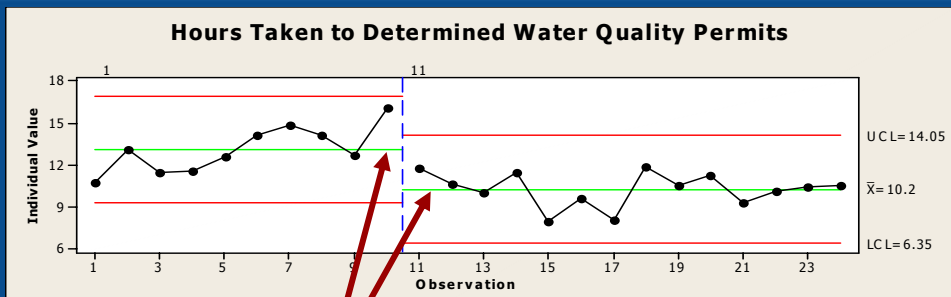


Highlights assignable causes

Trends in values by showing a shift in mean of values

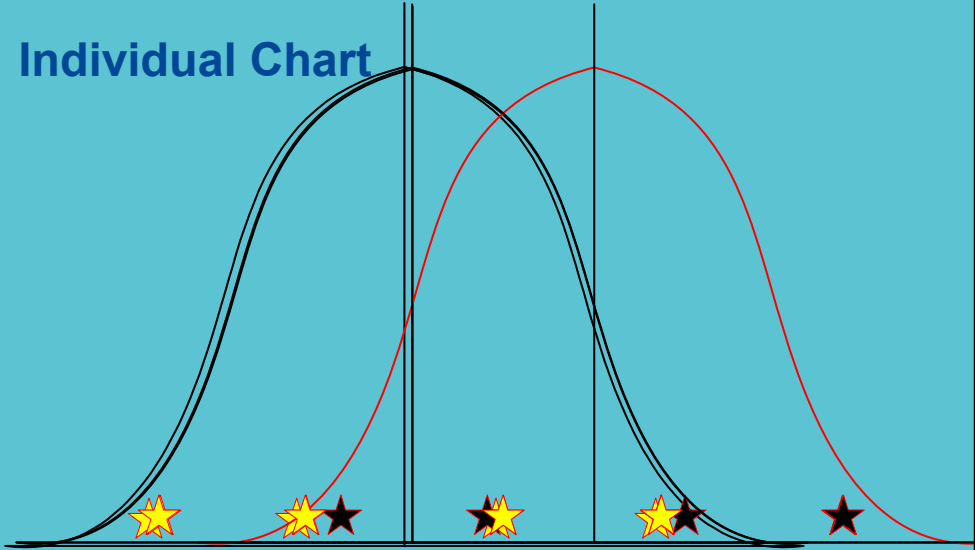
## Individual Chart - summary

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From previous example the shift in mean can be seen (after recalculating CLs and  $\bar{X}$  after signals)

## Individual Chart



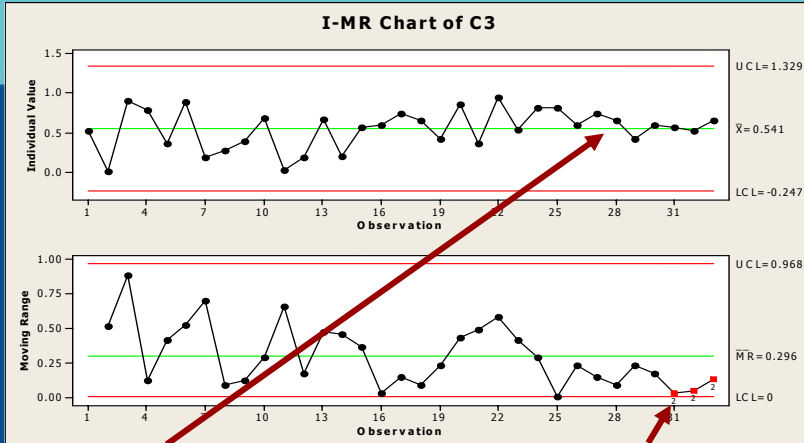
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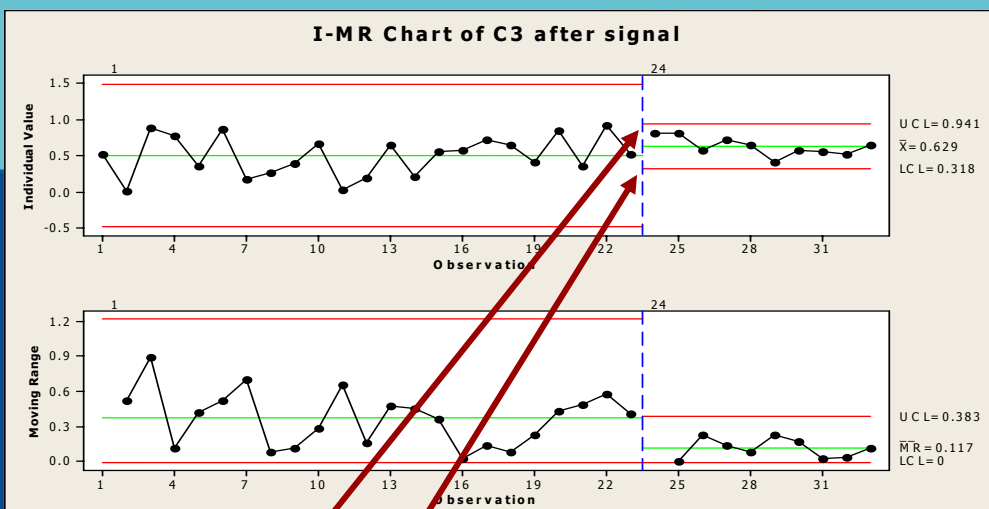
## Summary; Moving Range Chart

- Shows the difference between results and the average of that difference (known as R bar)
- Shows whether this difference is getting bigger or smaller to such an extent that it is statistically significant, ie not due to common cause variation
- Therefore shows changes in variation (spread of data) that is not always visible on an individuals chart



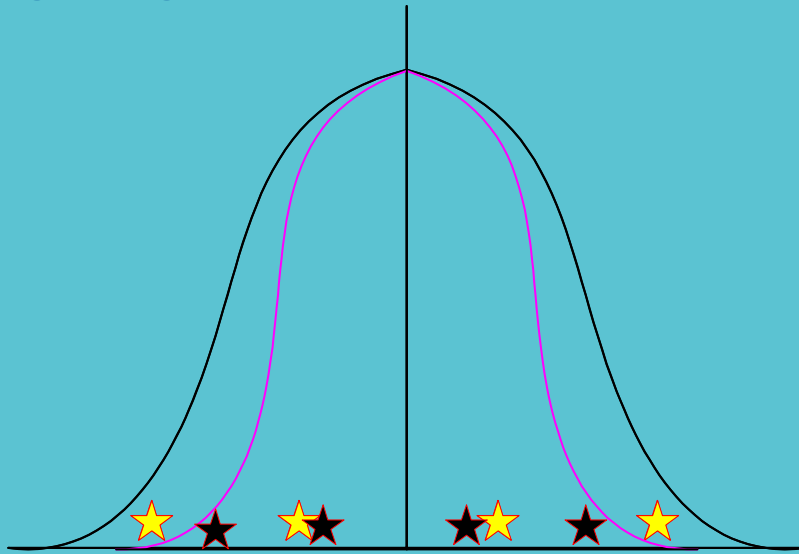
Therefore shows changes in variation (spread of data) that is not always visible on an individuals chart

Shows whether this difference is getting bigger or smaller to such an extent that it is statistically significant



The variation has reduced shown by the position of the new control limits

## Moving Range Chart



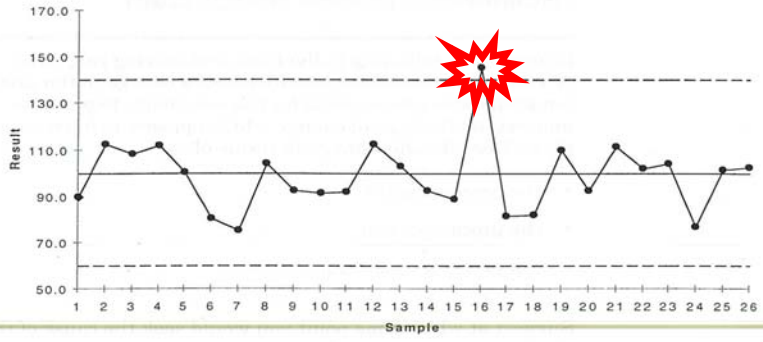
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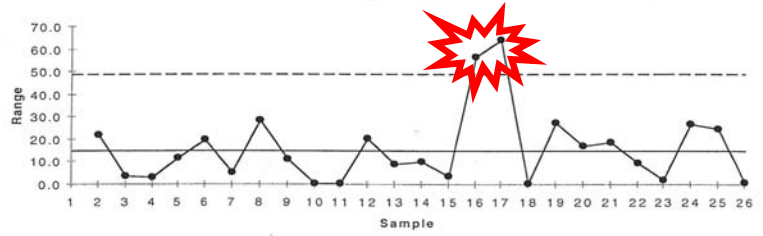
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## Exercise 2

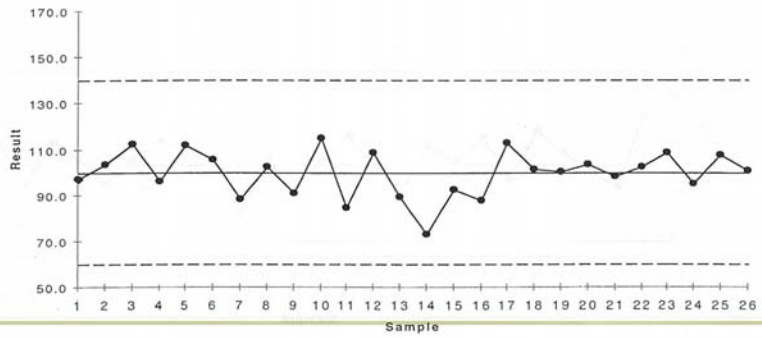
Example 1



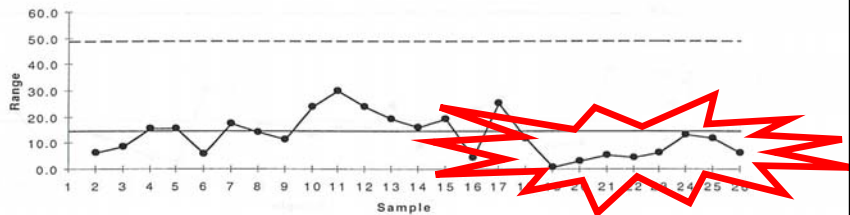
Moving Range



Example 2

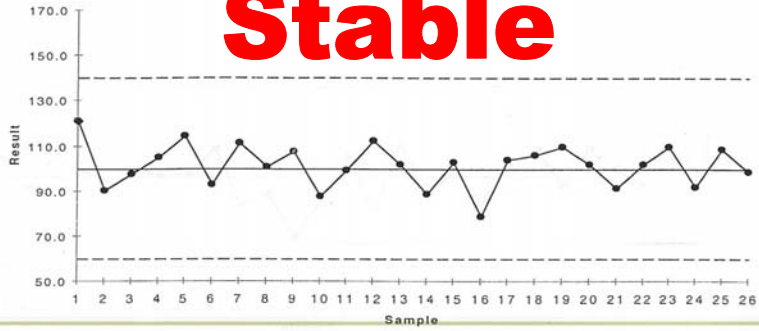


Moving Range

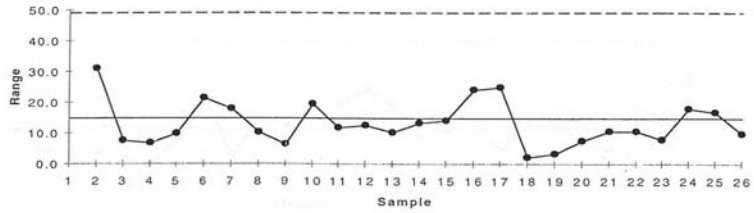


Example 3

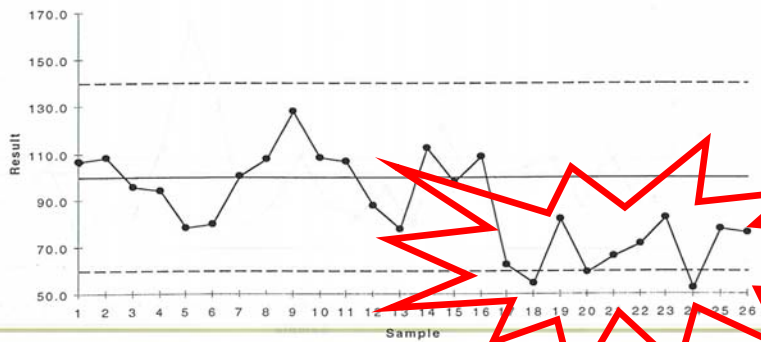
# Stable



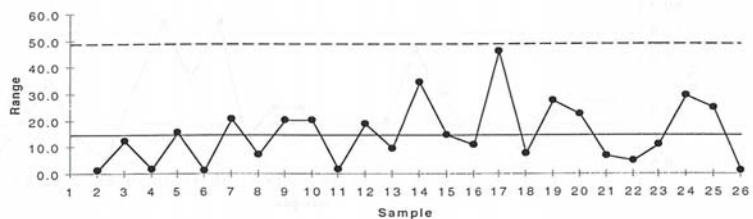
Moving Range

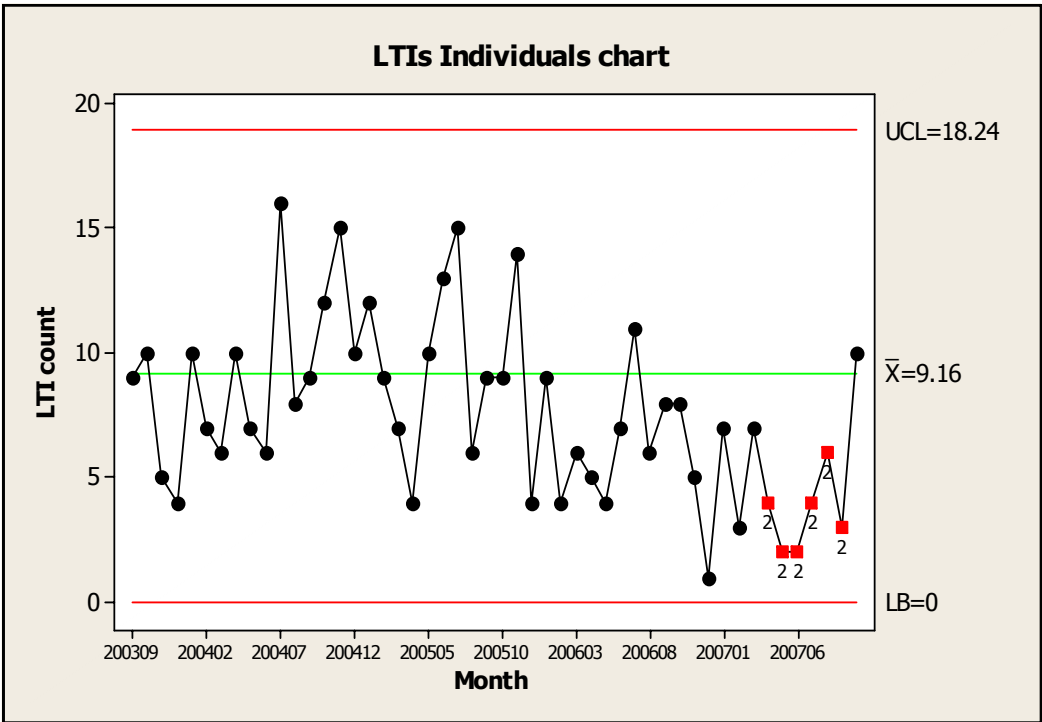
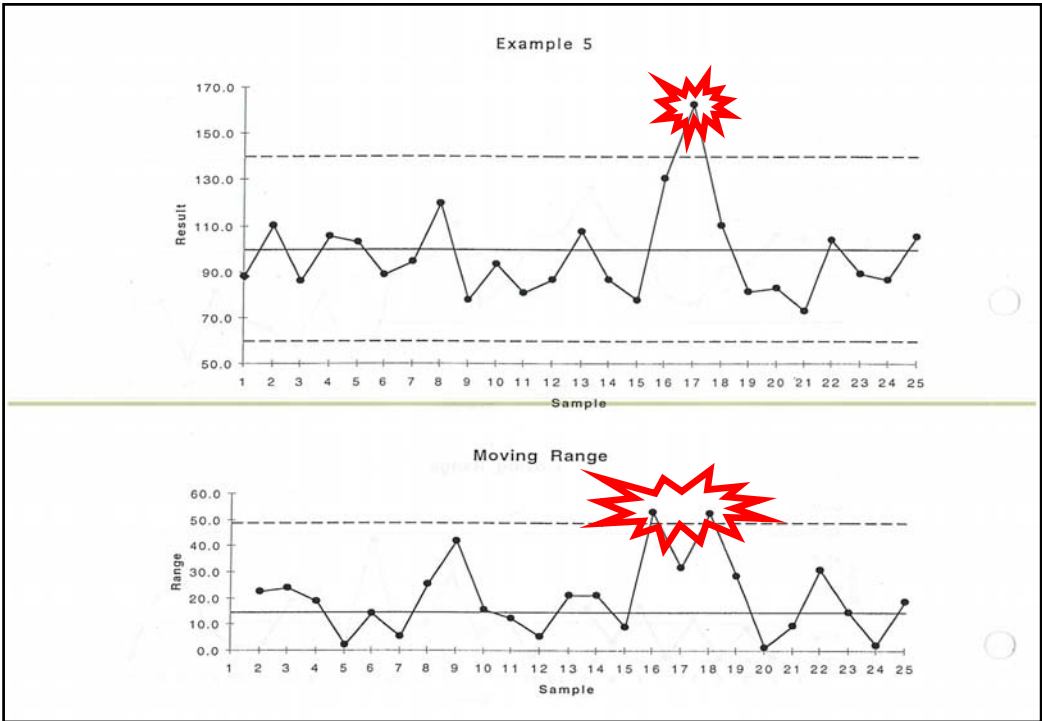


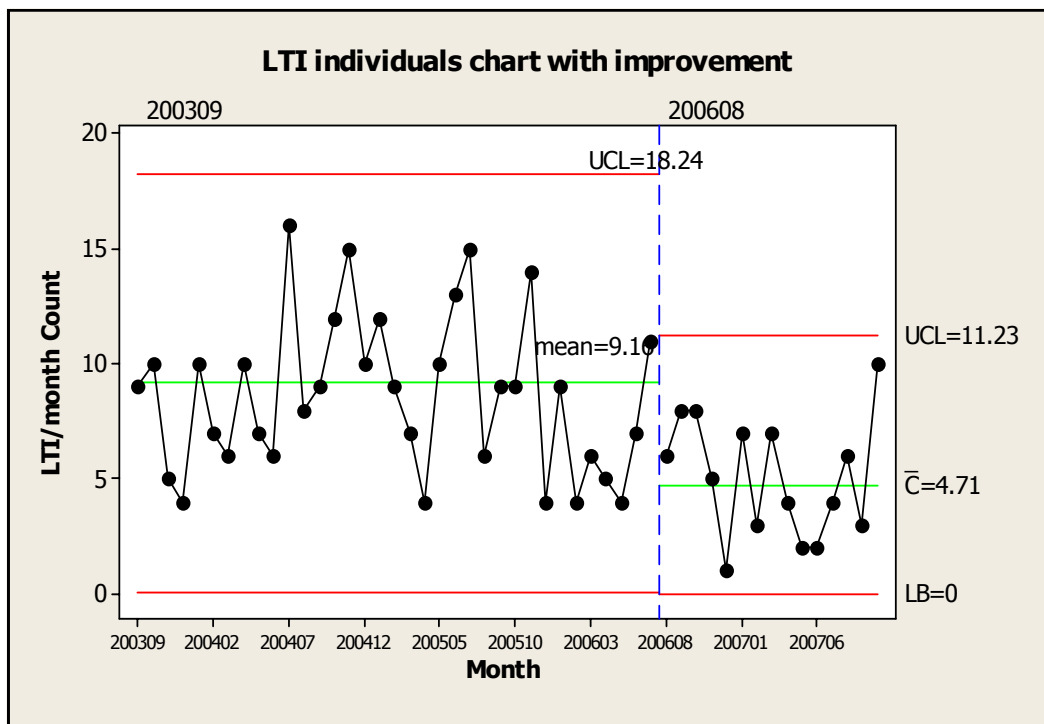
Example 4




Moving Range







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## Other types of charts (Xbar and R chart)

- Use them if there so much data that it is not possible to deal with as individual values
- Requires sampling of data
- E.g. pH of effluent in treatment recorded every minute

**Pros**

More sensitive to shifts in the centre of the process

**Cons**

Calculations more complex, need to ensure sampling is random

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THE END

Thank you and goodbye

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**Extra Slides if needed...**

## XBar R Charts and Subgroups

- a subgroup is a small sample of data (2-5) taken over a short interval of time from the same process.
  - Problem of random sampling
- The chart centre line is worked out as the average of the means of each sub group.
- The R values become the differences between each sub group mean

R is the range of the values within the sample group e.g.  $162-141=21$

Xbar is the mean of the sample  
e.g.  
 $(162+161+141)/3=154.7$

Xbar and R table							
observation ref	sample 1	sample 2	sample 3	sample 4	sample 5	sample 6	
1	162	161	147	154	143		
2	165	161	153	163	156	175	
3	171	141	166	147	164	166	
Xbar	170.3	154.7					
R	10	21					



## XBar R Charts and Subgroups

Xbar and R table						
observation ref	sample 1	sample 2	sample 3	sample 4	sample 5	sample 6
1	175	162	169	147	154	143
2	165	161	153	163	156	175
3	171	141	166	147	164	166
Xbar	170.3	154.7	162.7	152.3	158.0	161.3
R	10	21	16	16	10	32



## XBar R Charts and Subgroups Formulae

- Step 1 determine the centre line for each chart:

- **Xbar Chart**

Centre line = mean of the Xbar values

- $$= (170.3 + 154.7 + 162.7 + 152.3 + 158.0 + 161.3) / 6$$
$$= 159.88$$

- **R Chart**

Centre line (Rbar) = mean of the R values

- $$= (10 + 21 + 16 + 16 + 10 + 32) / 6$$
$$= 17.5$$

2

## XBar R Charts and Subgro

$A_2$  is a constant depending on sample size. For sample size of 3  
 $A_2=1.02$

- **Xbar Chart**
- **Step 2 determine the UCL**
- $UCL = \text{mean of the Xbar values} + A_2 \times Rbar$
- $UCL = 159.88 + 1.02 \times 17.5$
- $UCL = 177.75$
- **Step 3 determine the LCL**
- $LCL = \text{mean of the Xbar values} - A_2 \times Rbar$
- $LCL = 159.88 - 1.02 \times 17.5$
- $LCL = 142.05$

2

## XBar R Charts and Subgro

$A_2$  is a constant depending on sample size. For sample size of 3  
 $A_2=1.02$

- **Xbar Chart**
- **Step 2 determine the UCL**
- $UCL = \text{mean of the Xbar values} + A_2 \times Rbar$
- $UCL = 159.88 + 1.02 \times 17.5$
- $UCL = 177.75$
- **Step 3 determine the LCL**
- $LCL = \text{mean of the Xbar values} - A_2 \times Rbar$
- $LCL = 159.88 - 1.02 \times 17.5$
- $LCL = 142.05$

# XBar R Charts

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## Values of $A_2$

Sample Size n	2	3	4	5	6	7	
Value $A_2$	1.88	1.02	0.73	0.58	0.48	0.42	

## XBar R Charts and Subgroups

- **R Chart**
- **Step 4 determine the UCL**
- $UCL = D_4 \times Rbar$
- $UCL = 2.57 \times 17.5$
- $UCL = 44.975$
- **Step 3 determine the LCL**
- $LCL = D_3 \times Rbar$
- $LCL = 0 \times 17.5$
- $LCL = 0$

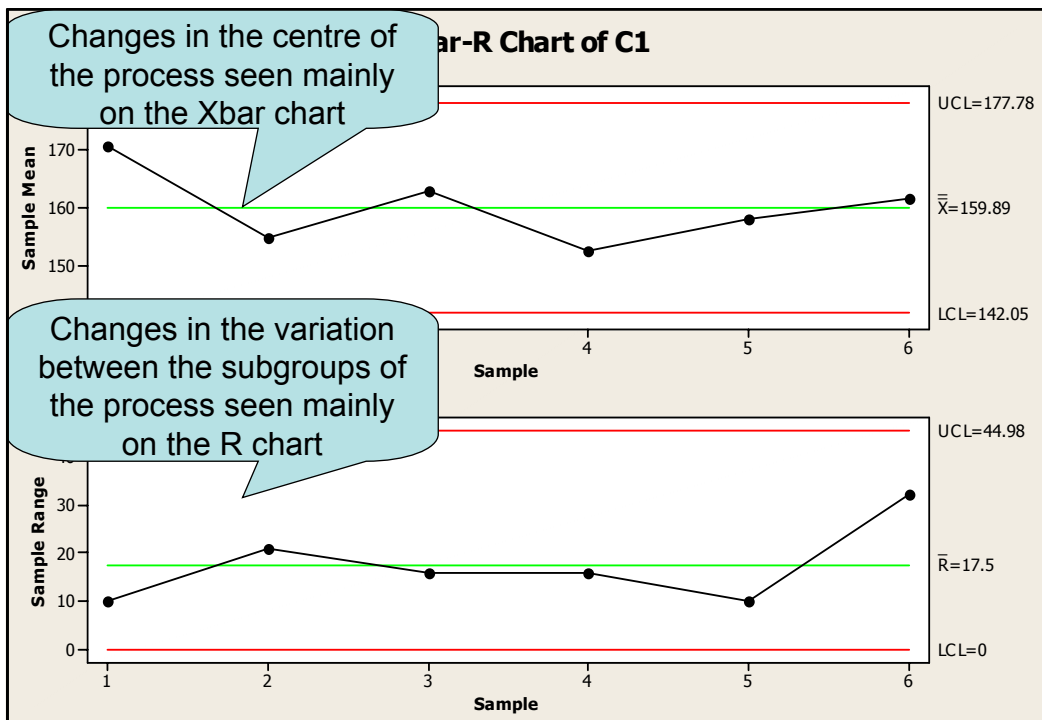
$D_4$  is a constant depending on sample size. For sample size of 3  $D_4=2.57$

$D_3$  is a constant depending on sample size. For sample size of 3  $D_3=0$



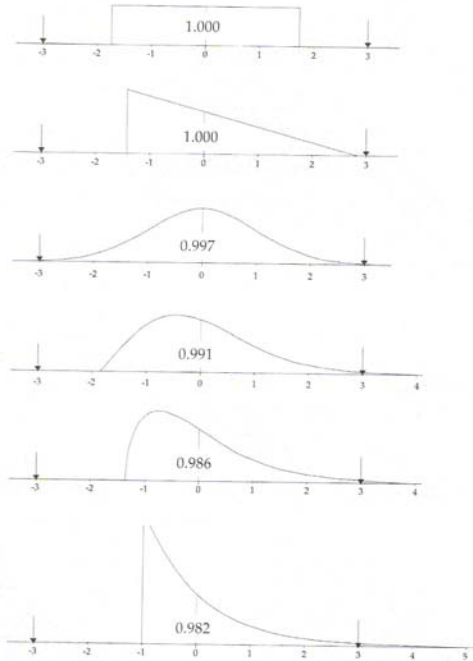
## Values of $D_4$ and $D_3$

Sample Size n	2	3	4	5	6	7
$D_4$	3.27	2.57	2.28	2.11	2.00	1.92
$D_3$	0	0	0	0	0	0.08



## Myths

- 1) Data must be normally distributed before a control chart can be used



## Myths

- 2) Individual data must be independent and have no auto-correlation with other collected data

Example: Wear on a cutting tool.

- Each part made causes wear on the cutting tool, so the last part made is always related to the previous one.

- 3) Data must be in control before a control chart can be used.

- Control charts are designed to identify the lack of control in a process

# Log normal vs normal

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